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Generalized Second Law of Thermodynamics in Extended Theories of Gravity

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Abstract By employing the general expression for temperature $T_h = |\kappa|/2\pi = \frac{1}{2\pi\tilde{r}_A}(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A})$ associated with the apparent horizon of FRW universe and assuming a region of an expanding universe enclosed by the apparent horizon as a thermal system in equilibrium, we are able to show that the generalized second law of thermodynamics holds in Gauss-Bonnet and more general Lovelock gravities.

Keywords Apparent horizon · Generalized second law · Extended theories of gravity

1 Introduction

It is possible to associate the notions of temperature and entropy with the apparent horizon of FRW universe analogous to the Hawking temperature and entropy associated with the black hole horizon [1–3]. The thermodynamical properties of the horizons have been studied in various theories of gravity (see for examples [4–14]) and its extension to the braneworld cosmology has also been made at the apparent horizon [15–18]. More recently, Cai et al. [6] showed by employing Clausius relation $\delta Q = T_h dS_h$ to the apparent horizon of a FRW universe that the modified Friedmann equations can be derived from the quantum corrected entropy-area relation. Since the extra higher derivative terms in Gauss-Bonnet and Lovelock gravities can be considered as a quantum corrections to the Einstein gravity which lead to suggest that the thermodynamic interpretation of field equations at horizon remains valid even if one includes the possible quantum corrections to the Einstein gravity. Therefore it is interesting to investigate generalized second law in these gravities. The Generalized Second Law of thermodynamics has been studied in various theories of gravities (see for examples [19–25]). More recently, by applying the general expression of temperature associated

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with the apparent horizon of FRW universe, it has been shown [26] that the generalized second law holds in the Einstein gravity. Hence, it is important to investigate whether the generalized law still holds at the apparent horizon even if one includes the possible quantum corrections to the Einstein gravity where the entropy-area relation no longer holds. In this paper, we will discuss this issue in case of Gauss-Bonnet and more general lovelock gravities. The paper is organized as follows. In the next section we will discuss generalized second law of thermodynamics for the Gauss-Bonnet gravity at apparent horizon of FRW universe. In Sect. 3, we will discuss generalized second law of thermodynamics in the case of Lovelock gravity. The conclusion and discussion are included in Sect. 4.

2 Gauss-Bonnet Gravity

Let us start with a $(n+1)$-dimensional FRW universe of metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_{n-1}^2\right), \tag{1}$$

where $d\Omega_{n-1}^2$ stands for the line element of $(n-1)$-dimensional unit sphere and spatial curvature constant $k = +1$, 0 and -1 represents to a closed, flat and open universe, respectively. The above metric (1) can be rewritten in spherical form

$$ds^2 = h_{ab} dx^a dx^b + \tilde{r}^2 d\Omega_{n-1}^2, \tag{2}$$

where $\tilde{r} = a(t)r$, $x^0 = t$, $x^1 = r$ and $h_{ab} = \mathrm{diag}(-1, \frac{a^2}{1-kr^2})$. One can work out the dynamical apparent horizon using $h^{ab}\partial_a \tilde{r}\partial_b \tilde{r} = 0$ which implies

$$\frac{1}{\tilde{r}_A^2} = H^2 + k/a^2, \tag{3}$$

where $H \equiv \frac{\dot{a}}{a}$ is the Hubble parameter and the over dot denotes derivative with respect to the cosmic time. It has been found that the entropy S associated with apparent horizon is proportional to the horizon area that satisfies the so-called area formula $S = A/4G$ [27], however it is well recognized that the area formula of entropy no longer holds in higher derivative gravities. So it is interesting to investigate whether, the generalized second law $\dot{S}_h + \dot{S}_m \geq 0$ still holds for a region of an expanding universe enclosed by apparent horizon if one considers higher derivative gravities. Where S_h and S_m are the entropies associated with the horizon and the source respectively. Let us first consider Gauss-Bonnet gravity which contains special combination of curvature-squared term, added to the Einstein-Hilbert action. The equations of motion for Gauss-Bonnet gravity are given by

$$G_{\mu\nu} + \alpha H_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{4}$$

where α is a constant with dimension $(length)^2$ and in case of superstring theory of low energy limit, α is regarded as the inverse string tension and is positive definite. $G_{\mu\nu}$ and $T_{\mu\nu}$ are the Einstein tensor and stress-energy tensor of the matter fields respectively and $H_{\mu\nu}$ is given by

$$H_{\mu\nu} = 2(RR_{\mu\nu} - 2R_{\mu\lambda}R_{\nu}^{\lambda} - 2R^{\gamma\delta}R_{\gamma\mu\delta\nu} + R_{\mu}^{\alpha\gamma\delta}R_{\alpha\nu\gamma\delta}) - \frac{1}{2}g_{\mu\nu}R_{GB}, \tag{5}$$

where $R_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$ is the Gauss-Bonnet term. In the vacuum Gauss-Bonnet gravity with/without a cosmological constant, static black hole solutions have been found and thermodynamics associated with the black hole horizons have been investigated in [28–30]. In this theory static spherically symmetric black hole solution is given by [1]

$$ds^2 = -e^{\lambda(r)}dt^2 + e^{\nu(r)}dr^2 + r^2 d\Omega_{n-1}^2, \tag{6}$$

with

$$e^{\lambda(r)} = e^{-\nu(r)} = 1 + \frac{r^2}{2\bar{\alpha}}\left(1 - \sqrt{1 + \frac{64\pi G\bar{\alpha}M}{n(n-1)\Omega_n r^n}}\right), \tag{7}$$

where $\bar{\alpha} = (n-2)(n-3)\alpha$ and M is the mass of black hole. As $\alpha \to 0$ in (4), the above metric leads to Schwarzschild metric in Einstein gravity. The entropy [28–30] associated with this black hole horizon is of the form, $S = \frac{A}{4G}(1 + \frac{(n-1)2\bar{\alpha}}{(n-3)r_+^2})$, where $A = n\Omega_n r_+^{n-1}$ is the horizon area and r_+ is the horizon radius of the black hole. It has been argue in [1] that the above entropy formula for black hole horizon also holds for the apparent horizon of FRW universe in Gauss-Bonnet gravity but replacing r_+ by the apparent horizon radius $\tilde{r}_A$ which turns out

$$S_h = \frac{A}{4G}\left(1 + \frac{(n-1)2\bar{\alpha}}{(n-3)\tilde{r}_A^2}\right), \tag{8}$$

where $A = n\Omega_n \tilde{r}_A^{n-1}$ is the horizon area. The Friedman equation in the Gauss-Bonnet gravity filled with perfect fluid of stress energy, $T_{\mu\nu} = (\rho + P)u_\mu u_\nu + Pg_{\mu\nu}$, is given by

$$\left(H^2 + \frac{k}{a^2}\right) + \bar{\alpha}\left(H^2 + \frac{k}{a^2}\right)^2 = \frac{16\pi G}{n(n-1)}\rho. \tag{9}$$

In terms of horizon radius, the above equation yields

$$\frac{1}{\tilde{r}_A^2} + \bar{\alpha}\frac{1}{\tilde{r}_A^4} = \frac{16\pi G}{n(n-1)}\rho. \tag{10}$$

Now, differentiating the above equation with respect to cosmic time, one gets

$$\dot{\tilde{r}}_A = \frac{8\pi G}{(n-1)}\frac{\tilde{r}_A^3 H(\rho + P)}{(1 + 2\bar{\alpha}/\tilde{r}_A^2)}, \tag{11}$$

where the continuity equation $\dot{\rho} = -nH(\rho + P)$ has been used. It can be seen from the above equation that $\dot{\tilde{r}}_A > 0$ in the expanding universe provided dominant energy condition hold. When $\alpha \to 0$, the above reduces to the result obtained in Einstein gravity. Now we turn to define horizon temperature which is proportional to surface gravity $\kappa = \frac{1}{2\sqrt{-h}}\partial_a(\sqrt{-h}h^{ab}\partial_b\tilde{r})$ and is given by

$$T_h = |\kappa|/2\pi = \frac{1}{2\pi\tilde{r}_A}\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right), \tag{12}$$

where $\frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} \leq 1$ ascertains that the temperature T_h is positive. Note that the temperature T_h does not depend on the gravity theory but depends [31] only on the background geometry. Thus the above expression for temperature also holds in Gauss-Bonnet gravity. Let us now consider a region of an expanding universe filled with perfect fluid of energy density

ρ and pressure P and is enveloped by the apparent horizon. Since the apparent horizon is not constant but varies with time. As the apparent radius changes, the volume enveloped by the apparent horizon will also change, however the thermal system bounded by the apparent horizon remains in equilibrium when it moves from one state to another so that the temperature associated with the horizon must be uniform and the same as the temperature of its surroundings. This requires that the temperature of the source inside the apparent horizon should be in equilibrium with the temperature associated with the horizon. Hence we have $T_m = T_h$, where T_m is the temperature of energy enclosed by the apparent horizon. Let the total energy of universe enclosed by the apparent horizon is the total matter energy $E = V\rho$, where $V = \Omega_n \tilde{r}_A^n$ is the volume enclosed by the apparent horizon, where $\Omega_n = \pi^{n/2}/\Gamma(n/2+1)$ is the volume of an n-dimensional unit ball. Let us now turn to find out, $T_h dS_h = \frac{1}{2\pi\tilde{r}_A}(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A})d(\frac{n\Omega_n\tilde{r}_A^{n-1}}{4G}(1 + \frac{(n-1)2\tilde{\alpha}}{(n-3)\tilde{r}_A^2}))$, which on simplifying yields

$$T_h \dot{S}_h = n\Omega_n \tilde{r}_A^n (\rho + P) H \left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right). \tag{13}$$

It is clear from (12) that the term, $(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}) > 0$, is positive. However, in the phase of accelerated expansion of the universe, $\rho + P$ may negative indicating that the second law $\dot{S}_h > 0$ does not hold in the expanding universe. But we will show that in the case of generalized second law this is not the case. It is well known that the Friedmann equations at apparent horizon of FRW universe satisfied [3, 15, 16, 32, 33] the thermal identity $dE = T_h dS_h + W dV$ instead of satisfying the standard first law $T_h dS_h = dE + P dV$. However the entropy S_m of the source inside the apparent horizon can be connected to its energy $E_m = V\rho$ and pressure P at the horizon by the first law of thermodynamics, $d(V\rho) = T_m dS_m - P dV$, where T_m is the temperature of the energy inside the horizon. As we have assumed the local equilibrium which implies $T_m = T_h$. Hence from the first law $dE_m = T_h dS_m - P dV$, one can obtain

$$T_h \dot{S}_m = n\Omega_n \tilde{r}_A^{n-1} \dot{\tilde{r}}_A (\rho + P) - n\Omega_n \tilde{r}_A^n H(\rho + P). \tag{14}$$

Adding (13) and (14), one gets

$$T_h(\dot{S}_m + \dot{S}_h) = \left(\frac{n}{2}\Omega_n \tilde{r}_A^{n-1}\right)(\rho + P)\dot{\tilde{r}}_A. \tag{15}$$

The right side of the above equation contains two factors. The first factor $\frac{n}{2}\Omega_n\tilde{r}_A^{n-1} = \frac{1}{2}A$, where A is the horizon area, is positive. And the second factor $(\rho + P)\dot{\tilde{r}}_A = \frac{8\pi G}{(n-1)} \frac{\tilde{r}_A^3 H(\rho+P)^2}{(1+2\tilde{\alpha}/\tilde{r}_A^2)}$ is also positive in the expanding universe. Hence the above equation $(\dot{S}_m + \dot{S}_h) \geq 0$ which implies that the generalized second law holds for a Gauss-Bonnet gravity in a region of expanding universe enclosed by the apparent horizon.

3 Lovelock Gravity

Let us now turn to derive the generalized second law in the case of lovelock gravity. The lovelock gravity generalizes the Einstein gravity when spacetime has a dimension greater than four. In this case the most general Lagrangian [34] that gives second order equations for the metric, is the sum over the dimensionally extended Euler density $L = \sum_{n=0}^{m} c_n L_n$, where c_n are arbitrary constants and L_n is the Euler density of a $2n$-dimensional manifold

$L_n = 2^{-n}\delta^{\mu_1\nu_1\dots\mu_n\nu_n}_{\alpha_1\beta_1\dots\alpha_n\beta_n} R^{\alpha_1\beta_1}_{\mu_1\nu_1}\dots R^{\alpha_n\beta_n}_{\mu_n\nu_n}$, where the generalized delta function $\delta^{\mu_1\nu_1\dots\mu_n\nu_n}_{\alpha_1\beta_1\dots\alpha_n\beta_n}$ is totally antisymmetric in both sets of indices and $R^{\alpha\beta}_{\gamma\delta}$ are the components of the curvature tensor. The static spherically symmetric black hole solutions can be obtained [35–38] in this theory. For an $(n+1)$-dimensional static spherically symmetric black hole of metric

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2_{n-1}, \tag{16}$$

with metric function is given by $f(r) = 1 - r^2 F(r)$, where $F(r)$ is determined by solving for real roots of mth-order polynomial equation, $\sum_{i=0}^{m}\hat{c}_i F^i(r) = \frac{16\pi GM}{n(n-1)\Omega_n} r^n$. Where M is the mass of the black hole and the coefficients $\hat{c}_i$ are $\hat{c}_0 = \frac{c_0}{n(n-1)}$, $c_1 = 1$ and $\hat{c}_i = c_i\prod_{j=3}^{2m}(n+1-j)$, for $i > 1$. The entropy [39] associated with the black hole horizon of lovelock gravity can be expressed as $S = \frac{A}{4G}\sum_{i=1}^{m}\frac{i(n-1)}{n-2i+1}\hat{c}_i r_+^{2-2i}$, where $A = n\Omega_n r_+^{n-1}$ is the horizon area and r_+ is the horizon radius. The above entropy formula of black hole horizon also holds for the apparent horizon of FRW universe and the entropy associated with the apparent horizon has the same expression but the black hole horizon radius r_+ is replaced by the apparent radius [1]. Hence the entropy expression for the apparent horizon turns out

$$S = \frac{A}{4G}\sum_{i=1}^{m}\frac{i(n-1)}{n-2i+1}\hat{c}_i\tilde{r}_A^{2-2i}. \tag{17}$$

The Friedman equation [1] in Lovelock theory of gravity is given by

$$\sum_{i=1}^{m}\hat{c}_i\left(H^2 + \frac{k}{a^2}\right)^i = \frac{16\pi G}{n(n-1)}\rho. \tag{18}$$

In terms of horizon radius $1/\tilde{r}_A^2 = H^2 + k/a^2$ the above (18) can be expressed as

$$\sum_{i=1}^{m}\hat{c}_i(\tilde{r}_A)^{-2i} = \frac{16\pi G}{n(n-1)}\rho. \tag{19}$$

From the above equation, one can obtain $\dot{\tilde{r}}_A$ by differentiation it with respect to cosmic time which yields

$$\dot{\tilde{r}}_A = \frac{8\pi G}{(n-1)}\frac{H(\rho+P)}{\sum_{i=1}^{m} i\hat{c}_i(\tilde{r}_A)^{-2i-1}}, \tag{20}$$

which is positive in the expanding universe provided the dominant energy condition holds. Again we assume the region of an expanding universe enclosed by the apparent horizon which remains in equilibrium when it moves from one state to another. This requires that the temperature associated with the apparent horizon is in equilibrium with the temperature of the energy enclosed by the boundary of the system. Also the expression for the temperature $T_h = |\kappa|/2\pi = \frac{1}{2\pi\tilde{r}_A}(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A})$ associated with the apparent horizon still holds in the lovelock gravity because the temperature associated with horizon depends on the background geometry but it is independent of the gravity theory. Let us now turn to find the expression for $T_h\dot{S}_h = \frac{1}{2\pi\tilde{r}_A}(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A})\frac{d}{dt}(\frac{n\Omega_n\tilde{r}_A^{n-1}}{4G}\sum_{i=1}^{m}\frac{i(n-1)}{n-2i+1}\hat{c}_i\tilde{r}_A^{2-2i})$ which on simplifying yields

$$T_h\dot{S}_h = n\Omega_n\tilde{r}_A^n(\rho+P)H\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right). \tag{21}$$

It has been shown that the entropy S_h and the temperature T_h associated with the apparent horizon of FRW universe together with the Friedmann equation satisfy [3, 15, 16] the thermal identity $dE = T_h dS_h + W dV$ in Einstein, Gauss-Bonnet and Lovelock theories of gravity. However the entropy S_m of the universe inside the apparent horizon can be connected to its energy $E_m = V\rho$ and pressure P at the horizon by the first law $d(V\rho) = T_m dS_m - P dV$ of thermodynamics. Since the thermal system bounded by the apparent horizon remains in equilibrium so that the temperature of the system must be uniform and the same as the temperature of its surroundings. Hence we have $T_m = T_h$. One can obtain from the first law ($dE_m = T_h dS_m - P dV$)

$$T_h \dot{S}_m = n\Omega_n \tilde{r}_A^{n-1} \dot{\tilde{r}}_A(\rho + P) - n\Omega_n \tilde{r}_A^n H(\rho + P). \tag{22}$$

Adding (21) and (22), one gets

$$T_h(\dot{S}_m + \dot{S}_h) = \left(\frac{n}{2}\Omega_n \tilde{r}_A^{n-1}\right)(\rho + P)\dot{\tilde{r}}_A. \tag{23}$$

The right side of the above equation contains two factors. The first factor $\frac{n}{2}\Omega_n \tilde{r}_A^{n-1} = \frac{1}{2}A$, where A is the horizon area, is positive. And the second factor $(\rho + P)\dot{\tilde{r}}_A = \frac{8\pi G}{(n-1)} \frac{H(\rho+P)^2}{\sum_{i=1}^m i\hat{c}_i(\tilde{r}_A)^{-2i-1}}$ is also positive in the expanding universe. Hence from the above equation (23), the generalized second law $(\dot{S}_m + \dot{S}_h) \geq 0$ holds for a Lovelock gravity in a region enclosed by the apparent horizon of FRW universe.

4 Conclusion

In this paper we studied the generalized second law of thermodynamics in the expanding universe for the Gauss-Bonnet and Lovelock gravities. It is shown by employing the general expression of the temperature $T = |\kappa|/2\pi = \frac{1}{2\pi\tilde{r}_A}(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A})$ associated with the apparent horizon of FRW universe together with the assumption that the universe bounded by the apparent horizon is in equilibrium so that the temperature of the system must be uniform as the temperature of its surroundings, we are able to show that the generalized second law holds in the Gauss-Bonnet and Lovelock gravities. In this analysis, the general expression of temperature, $T_h = |\kappa|/2\pi = \frac{1}{2\pi\tilde{r}_A}(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A})$, has been used to discuss generalized second law in various gravity theories however, in the earlier work, an approximate horizon temperature $T_h = 1/2\pi\tilde{r}_A$ has been assumed to investigate the generalized second law (see for examples [19–25]). In these references, the temperature T_m of the source has been taken proportional to the horizon temperature T_h through $T_m = bT_h$, where $b > 0$ to assure that the temperature of the system is positive. When $b = 1$, we have $T_m = T_h$ which corresponds to a local equilibrium of the system.

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References

1. Cai, R.G., Kim, S.P.: J. High Energy Phys. **02**, 050 (2005). arXiv:hep-th/0501055
2. Gibbons, G.W., Hawking, S.W.: Phys. Rev. D **15**, 2738 (1977)
3. Akbar, M., Cai, R.G.: Phys. Rev. D **75**, 084003 (2007)

4. Eling, C., Guedens, R., Jacobson, T.: Phys. Rev. Lett. **96**, 121301 (2006)
5. Akbar, M., Cai, R.G.: Phys. Lett. B **648**, 243 (2007)
6. Cai, R.G., Cao, L.M., Hu, Y.P.: 0807.1232 [hep-th]
7. Cai, R.G., Cao, L.M.: Phys. Rev. D **75**, 064008 (2007)
8. Akbar, M., Cai, R.G.: Phys. Lett. B **635**, 7 (2006). arXiv:hep-th/0602156
9. Kothawala, D., Sarkar, S., Padmanabhan, T.: arXiv:gr-qc/0701002
10. Padmanabhan, T.: Class. Quantum Gravity **19**, 5387 (2002). arXiv:gr-qc/0204019
11. Padmanabhan, T.: Phys. Rep. **406**, 49 (2005). arXiv:gr-qc/0311036
12. Paranjape, A., Sarkar, S., Padmanabhan, T.: Phys. Rev. D **74**, 104015 (2006). arXiv:hep-th/0607240
13. Akbar, M., Siddiqui, A.A.: Phys. Lett. B **656**, 217 (2007)
14. Akbar, M.: Chin. Phys. Lett. **24**(5), 1158 (2007). hep-th/0702029
15. Cai, R.G., Cao, L.M.: hep-th/0612144
16. Sheykhi, A., Wang, B., Cai, R.G.: Nucl. Phys. B **779**, 1 (2007). hep-th/0701198
17. Sheykhi, A., Wang, B., Cai, R.G.: hep-th/0701261
18. Cai, R.G.: 0712.2142 [hep-th]
19. Wu, S.F., Wang, B., Yang, G.H., Zhang, P.M.: Class. Quantum Gravity **25**, 235018 (2008). 0801.2688 [hep-th]
20. Mohseni Sadjadi, H.: Phys. Lett. B **645**, 108 (2007)
21. Mohseni Sadjadi, H.: Phys. Rev. D **76**, 104024 (2007)
22. Barrow, J.D.: Nucl. Phys. B **310**, 743 (1988)
23. Giovannini, M.: Phys. Rev. D **59**, 121301 (1999)
24. Zimdahl, W., Pavon, D.: Phys. Rev. D **61**, 108301 (2000)
25. Setare, M.R.: Phys. Lett. B **641**, 130 (2006)
26. Akbar, M.: Chin. Phys. Lett. **25**, 4199 (2008). 0808.0169 [gr-qc]
27. Wald, R.M.: Phys. Rev. D **48**, 3427 (1993)
28. Cai, R.G.: Phys. Rev. D **65**, 084014 (2002)
29. Cai, R.G., Guo, Q.: Phys. Rev. D **69**, 104025 (2004)
30. Cai, R.G., Soh, K.S.: Phys. Rev. D **59**, 044013 (1999)
31. Wang, P.: Phys. Rev. D **72**, 024030 (2005)
32. Frolov, A.V., Kofman, L.: J. Cosmol. Astropart. Phys. **0305**, 009 (2003)
33. Bousso, R.: Phys. Rev. D **71**, 064024 (2005). arXiv:hep-th/0412197
34. Lovelock, D.: J. Math. Phys. **12**, 498 (1971)
35. Boulware, D.G., Deser, S.: Phys. Rev. Lett. **55**, 2656 (1985)
36. Wheeler, J.T.: Nucl. Phys. B **268**, 737 (1986)
37. Wheeler, J.T.: Nucl. Phys. B **273**, 732 (1986)
38. Myers, R.C., Simon, J.Z.: Phys. Rev. D **38**, 2434 (1988)
39. Cai, R.G.: Phys. Lett. B **582**, 237 (2004). hep-th/0311240